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On M5-branes in $\mathcal{N} = 6$ membrane action

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Abstract: In this note we study M5-branes in the multiple membrane action which is recently proposed by Aharony-Bergman-Jafferis-Maldacena. We write down the $\mathcal{N} = 6$ supersymmetry transformation of the action and obtain 1/2 BPS equations and their solutions. They are expected to represent membranes ending on a M5-brane. We also consider the M5-M2 bound state in the action.

Keywords: Brane Dynamics in Gauge Theories, Solitons Monopoles and Instantons, M-Theory, Chern-Simons Theories.

Contents

1. Introduction

Since an action of multiple M2-branes proposed by the Bagger and Lambert [1] (see for earlier works [2, 3]), it has been studied intensively [4]–[51]. Recently, a three dimensional $\mathcal{N} = 6$ supersymmetric Chern-Simons-matter conformal field theory with gauge group $\mathrm{U}(N) \times \mathrm{U}(N)$ was proposed as an action of the low energy limit of N M2-branes on $\mathbf{C}^4/\mathbf{Z}_k$ by Aharony-Bergman-Jafferis-Maldacena (ABJM) [52]. Many aspects of the theory have been studied [53]–[66].

The M5-branes are also interesting and still mysterious objects in M-theory. In this paper, we study the BPS equations of this ABJM action, which will describe the M5-brane. We find solutions of these equations. These BPS equations are analogues of the Basu-Harvey equation [2] and we expect that the solutions represent N M2-branes ending on the M5-brane.

We also expect that the flat M5-branes will be constructed from infinitely many M2-branes, as the D4-D2 bound state. This M5-M2 bound state has different supersymmetries from the ones which M5-branes have. Thus M5-M2 bound state on the orbifold will not be BPS and we can not expect that there is the BPS solution corresponding to this bound state in the ABJM action. Therefore, instead of the BPS equation, we will discuss solutions of the equations of motion, which will describe the M5-M2 bound state.

The organization of this paper is as follows. In section two we briefly review the ABJM action and present an manifest $\mathcal{N} = 6$ SUSY transformation of this action. In section three we study the BPS equations of the ABJM action and their solutions. The M5-M2 bound state is discussed in section four. In section five we draw conclusions and discuss future problems.

2. $\mathcal{N}=6$ SUSY action and SUSY transformation

In this section we will briefly review the ABJM action. The fields in the ABJM action are $\mathrm{U}(N)\times\mathrm{U}(N)$ gauge fields A_μ and $\hat{A}_\mu$, four $\mathrm{U}(N)\times\mathrm{U}(N)$ bi-fundamental bosonic fields Y^A and fermionic spinor fields ψ_A, where $A=1,2,3,4$.

The SU(4) invariant action of this theory is explicitly given by [53, 52]

$$S=\int d^3x\Big[\frac{k}{4\pi}\varepsilon^{\mu\nu\lambda}\mathrm{Tr}\left(A_\mu\partial_\nu A_\lambda+\frac{2i}{3}A_\mu A_\nu A_\lambda-\hat{A}_\mu\partial_\nu\hat{A}_\lambda-\frac{2i}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\lambda\right)$$
$$-\mathrm{Tr}D_\mu Y_A^\dagger D^\mu Y^A-i\mathrm{Tr}\,\psi^{A\dagger}\gamma^\mu D_\mu\psi_A-V_{\mathrm{bos}}-V_{\mathrm{ferm}}\Big] \tag{2.1}$$

with the potentials

$$V_{\mathrm{bos}}=-\frac{4\pi^2}{3k^2}\mathrm{Tr}\Big(Y^AY_A^\dagger Y^BY_B^\dagger Y^CY_C^\dagger+Y_A^\dagger Y^AY_B^\dagger Y^BY_C^\dagger Y^C$$
$$+4Y^AY_B^\dagger Y^CY_A^\dagger Y^BY_C^\dagger-6Y^AY_B^\dagger Y^BY_A^\dagger Y^CY_C^\dagger\Big), \tag{2.2}$$

and

$$V_{\mathrm{ferm}}=-\frac{2i\pi}{k}\mathrm{Tr}\Big(Y_A^\dagger Y^A\psi^{B\dagger}\psi_B-\psi^{B\dagger}Y^AY_A^\dagger\psi_B-2Y_A^\dagger Y^B\psi^{A\dagger}\psi_B+2\psi^{B\dagger}Y^AY_B^\dagger\psi_A$$
$$-\epsilon^{ABCD}Y_A^\dagger\psi_BY_C^\dagger\psi_D+\epsilon_{ABCD}Y^A\psi^{B\dagger}Y^C\psi^{D\dagger}\Big), \tag{2.3}$$

The convention of the spinors is same as in [53] and $k=8\pi K$.

The $\mathcal{N}=6$ SUSY transformation is given by

$$\begin{aligned}
\delta Y^A &= i\omega^{AB}\psi_B,\\
\delta Y_A^\dagger &= i\psi^{\dagger\,B}\omega_{AB},\\
\delta\psi_A &= -\gamma_\mu\omega_{AB}D_\mu Y^B+\frac{2\pi}{k}\left(-\omega_{AB}(Y^CY_C^\dagger Y^B-Y^BY_C^\dagger Y^C)+2\omega_{CD}Y^CY_A^\dagger Y^D\right),\\
\delta\psi^{A\dagger} &= D_\mu Y_B^\dagger\gamma_\mu\omega^{AB}+\frac{2\pi}{k}\left(-(Y_B^\dagger Y^CY_C^\dagger-Y_c^\dagger Y^CY_B^\dagger)\omega^{AB}+2Y_D^\dagger Y^AY_C^\dagger\omega^{CD}\right),\\
\delta A_\mu &= -\frac{2\pi}{k}(Y^A\psi^{B\dagger}\gamma_\mu\omega_{AB}+\omega^{AB}\gamma_\mu\psi_AY_B^\dagger),\\
\delta\hat{A}_\mu &= \frac{2\pi}{k}(\psi^{A\dagger}Y^B\gamma_\mu\omega_{AB}+\omega^{AB}\gamma_\mu Y_A^\dagger\psi_B),
\end{aligned} \tag{2.4}$$

where ψ and ω_{AB} have lower spinor indices, while $\psi^\dagger$ and ω^{AB} have upper spinor indices.

By the 6 Majorana (2+1)-dimensional spinors, ϵ_i $(i=1,\ldots,6)$, which are the $\mathcal{N}=6$ SUSY generators, the ω_{AB} is given by

$$\omega_{AB}=\epsilon_i(\Gamma^i)_{AB}, \tag{2.5}$$
$$\omega^{AB}=\epsilon_i((\Gamma^i)^*)^{AB}, \tag{2.6}$$

in which the A,B indices are anti-symmetric and we take 4 by 4 matrices Γ^i as follows:

$$\begin{aligned}
\Gamma^1&=\sigma_2\otimes 1_2, & \Gamma^4&=-\sigma_1\otimes\sigma_2,\\
\Gamma^2&=-i\sigma_2\otimes\sigma_3, & \Gamma^5&=\sigma_3\otimes\sigma_2,\\
\Gamma^3&=i\sigma_2\otimes\sigma_1, & \Gamma^6&=-i1_2\otimes\sigma_2,
\end{aligned} \tag{2.7}$$

which are chiral decomposed 6-dimensional Γ-matrices. These matrices satisfy

$$\{\Gamma^i, \Gamma^{j\dagger}\} = 2\delta_{ij}, \qquad\qquad (\Gamma^i)_{AB} = -(\Gamma^i)_{AB}, \tag{2.8}$$

$$\frac{1}{2}\epsilon^{ABCD}\Gamma^i_{CD} = -(\Gamma^{i\dagger})^{AB} = ((\Gamma^i)^*)^{AB}. \tag{2.9}$$

Therefore we have following relations[1]

$$(\omega^{AB})_\alpha = ((\omega_{AB})^*)_\alpha, \quad \omega^{AB} = \frac{1}{2}\epsilon^{ABCD}\omega_{CD}. \tag{2.10}$$

We can explicitly check that the action (2.1) is indeed invariant under the transformation (2.4). We can also check that if we restrict $\omega_{a\dot{b}} = 0$, $(a = 1, 2, \dot{b} = 3, 4)$, this transformation is same as the usual SUSY transformation of the $\mathcal{N} = 2$ superfield formalism [53]. Note that since the superfield is written in the Wess-Zumino gauge, the SUSY transformation is corrected by the super gauge transformation with the gauge parameter proportional to σ and $\tilde{\sigma}$. Including these, (2.4) will coincides with the usual supersymmetry transformation in the superspace.

3. M5-brane from the M2-brane action

We consider solutions of the BPS equation of the ABJM action which corresponds to the M2-branes ending on the M5-branes as in Basu-Harvey equation [2]. The BPS condition is $\delta\psi_A = 0$. Here we will assume $Y^3 = Y^4 = 0$ and $Y^1 = Y^1(x^2)$, $Y^2 = Y^2(x^2)$, namely the world-volume of the M5-branes are along $\{x^0, x^1, x^4, x^5, x^6, x^7\}$. We also assume

$$\gamma^2\omega_{12} = \omega_{12}, \;\; \gamma^2\omega_{34} = \omega_{34}, \;\; \gamma^2\omega_{a\dot{b}} = -\omega_{a\dot{b}}, \;\; \gamma^2\omega_{\dot{a}b} = -\omega_{\dot{a}b}, \tag{3.1}$$

where $a = 1, 2$ and $\ldots b = 3, 4$. Note that, for example, ω_{12} is a complex conjugate of ω_{34}. This means that we are considering a $\frac{1}{2}$ BPS solution, i.e. a solution with unbroken 6 supersymmetries. We expect this will be obtained from the M5-M2-brane on $\mathbf{R}^{10,1}$, which have unbroken 8 supersymmetries, by the $\mathbf{Z}_k$ orbifolding.

Then the SUSY transformation (2.4) for ψ becomes

$$0 = \frac{dY^1}{dx^2} + \frac{2\pi}{k}(Y^2 Y_2^\dagger Y^1 - Y^1 Y_2^\dagger Y^2), \tag{3.2}$$

$$0 = \frac{dY^2}{dx^2} + \frac{2\pi}{k}(Y^1 Y_1^\dagger Y^2 - Y^2 Y_1^\dagger Y^1), \tag{3.3}$$

which can be written as

$$\frac{dY^a}{dx^2} = -\frac{2\pi}{k}(Y^b Y_b^\dagger Y^a - Y^a Y_b^\dagger Y^b). \tag{3.4}$$

[1]if we consider the action given in [53] in which the sign of the last two terms in (2.3) are reversed, it still have $\mathcal{N} = 6$ SUSY by only replacing $\Gamma^i \to R\Gamma^i R$ where $R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$. It satisfies $\frac{1}{2}\epsilon^{ABCD}\Gamma^i_{CD} = (\Gamma^{i\dagger})^{AB} = -((\Gamma^i)^*)^{AB}$.

These equations have global U(2) invariance which acts on a, b indices and $\mathrm{U}(N) \times \mathrm{U}(N)$ gauge invariance.

As in [2], if we have $N \times N$ matrices S^a which satisfy

$$\begin{aligned} S^1 &= S^2 S^{2\dagger} S^1 - S^1 S^{2\dagger} S^2 \\ S^2 &= S^1 S^{1\dagger} S^2 - S^2 S^{1\dagger} S^1, \end{aligned} \tag{3.5}$$

then

$$Y^a = \sqrt{\frac{k}{4\pi x^2}} S^a, \tag{3.6}$$

($x^2 > 0$) is the BPS solution represents N M2-brane ending on a M5-brane.[2] Instead of (3.6),

$$Y^a = f^a(x^2) S^a, \tag{3.7}$$

with

$$\frac{df^1}{dx^2} + \frac{1}{2}|f^2|^2 f^1, \;\; \frac{df^2}{dx^2} + \frac{1}{2}|f^1|^2 f^2, \tag{3.8}$$

is also a solution, which has a non-trivial real modulus, We can assume without loss of generality that f^i are real. Then, $C_0 \equiv |f^1|^2 - |f^2|^2$ is a constant and we obtain

$$\frac{d(f^2)^2}{dx^2} + \frac{1}{4}(f^2)^2((f^2)^2 + C_0) = 0, \tag{3.9}$$

which has a solution modulo the translation.

For $N = 2$, we have the following explicit solution of (3.5),

$$\begin{aligned} S^1 &= \frac{1}{2}(\sigma_1 + i\sigma_2), \\ S^2 &= \frac{1}{2}(1_2 - \sigma_3). \end{aligned} \tag{3.10}$$

This solution seems strange as a fuzzy 3-sphere because S^2 is Hermite and diagonalized matrix, thus it might not repersent an object extends in three directions.[3] However, we note that S^a is in a bi-fundamental representation, instead of an adjoint representation and there are $\mathrm{U}(N) \times \mathrm{U}(N)$ gauge symmetry, instead of $\mathrm{U}(N)$. Therefore, we can always diagonalize S^2 and this solution may represent a fuzzy 3-sphere. For arbitrary N, by the $\mathrm{U}(N) \times \mathrm{U}(N)$ gauge symmetry, we can take

$$(S^2)_{ij} = \alpha_i \delta_{ij}, \tag{3.11}$$

where α^i is real and non-negative number. We can further assume $\alpha_{i+1} \leq \alpha_i$ without loss of generality. Then, from the first equation of (3.5), we see that $(S^1)_{ij} = 0$ if $(\alpha_i)^2 - (\alpha_j)^2 = 1$. This implies S^1 is block diagonalized if $(\alpha_{i+1})^2 = (\alpha_i)^2 - 1$ is not satisfied for any

[2] $Y^a = \sqrt{\frac{16\pi k}{-x^2}} S^a$ ($x^2 < 0$) with $S^1 = S^1 S^{2\dagger} S^2 - S^2 S^{2\dagger} S^1$ and $S^2 = S^2 S^{1\dagger} S^1 - S^1 S^{1\dagger} S^2$ is also a BPS solution and represents an anti-M5-brane.

[3] We thank S. Kawai and S. Sasaki for discussing this point.

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$i = 1, \ldots, N-1$. The block diagonalized S^1 will represent several M5-branes. Thus, we assume $(\alpha_{i+1})^2 = (\alpha_i)^2 - 1$, then

$$(S^1)_{ij} = \beta_i\, \delta_{i,\,j-1} \;\; (i, j = 1, \ldots, N). \tag{3.12}$$

If we set $\beta_N = 0$ and $\beta_0 = 0$ for convenience, we can write $(S^1(S^1)^\dagger)_{ij} = \delta_{ij}(\beta_i)^2$ and $((S^1)^\dagger S^1)_{ij} = \delta_{ij}(\beta_{i-1})^2$. Now we can easily solve the second equation of (3.5),

$$((\beta_i)^2 - (\beta_{i-1})^2)\alpha_i = \alpha_i, \;\; (i = 1, \ldots N). \tag{3.13}$$

Indeed, this implies that $\alpha_N = 0$ for $i = N$ and $\beta_1 = 1$ for $i = 1$. (Here we have assumed S^1 is not block diagonalized.) Therefore, we find the BPS solution representing the N M2-branes ending on a M5-brane is (3.6) with

$$(S^1)_{ij} = \delta_{i,\,j-1}\sqrt{i}, \;\; (S^2)_{ij} = \delta_{ij}\sqrt{N-i} \;\; (i, j = 1, \ldots, N). \tag{3.14}$$

Of course, a diagonal sum of (3.14) is also a BPS solution.[4]

We can estimate the tension of the M5-brane. In the large N limit, the approximate radius of the fuzzy 3-sphere is $r \sim \sqrt{kN/(4\pi x^2)}$. The action is evaluated as

$$S \sim -2\int d^3x \mathrm{Tr} D_\mu Y_a^\dagger D^\mu Y^a \sim -2\int d^3x \frac{k}{16\pi (x^2)^3} \mathrm{Tr}(S^a (S^a)^\dagger) \sim -\int dx^0 dx^1 dr r^3 \frac{2\pi}{k}, \tag{3.15}$$

and the area of the three dimensional sphere $2\pi^2$ should be divided by k because of the $\mathbf{Z}_k$ orbifolding. Thus, the tension of the M5-brane is independent of k and N as expected.

For the fuzzy 2-sphere in D1-branes ending on D3-branes, we can obtain the non-commutative R^2 by taking a limit which corresponds to focusing on the north pole of the fuzzy 2-sphere. We will consider a similar limit for our fuzzy 3-sphere. The equations (3.5) can be written by four Hermite matrices as

$$\begin{aligned} A &= i\left([B, C^2 + D^2] + \{A, [C, D]\}\right), \\ B &= i\left(-[A, C^2 + D^2] + \{B, [C, D]\}\right), \\ C &= i\left([D, A^2 + B^2] + \{C, [A, B]\}\right), \\ D &= i\left(-[C, A^2 + B^2] + \{D, [A, B]\}\right), \end{aligned} \tag{3.16}$$

where

$$S^1 = A + iB, \quad S^2 = C + iD. \tag{3.17}$$

We assume $A = \Lambda + \delta$, where $\Lambda \gg 1$ is a constant, and $B = 0$. Then (3.16) becomes

$$[C, D] = -\frac{i}{2}, \quad C = i[D, 2\Lambda\delta], \;\; D = -i[C, 2\Lambda\delta], \;\; [\delta, C^2 + D^2] = 0, \tag{3.18}$$

which can be solved as

$$C = \frac{1}{\sqrt{2}}\hat{p}, \;\; D = \frac{1}{\sqrt{2}}\hat{q}, \;\; -4\Lambda\delta = \hat{p}^2 + \hat{q}^2 + const. \tag{3.19}$$

In the limit which take the M2-branes to D2-branes [5, 10, 52], B is the compactified direction and C and D span the non-commutative 2-plane.

[4] This solution was obtained also in [71].

4. M5-branes with flux

The M5-brane with flux can be considered as the bound state of M2-branes and M5-branes. We expect that there are solitonic solutions in the action (2.1) which represent the bound states. Because the M5-brane extending in $\{x^0, x^1, x^2\}$ and three directions in $C^4/\mathbf{Z}_k$, the supersymmetries will be completely broken. Actually, the action does not have additional non-linearly realized supersymmetry which would restore supersymmetry. Therefore, we will study the equations of motion, instead of BPS equations.

First, by the $2N \times 2N$ Hermitian matrices

$$\tilde{Y}_A = \begin{pmatrix} 0 & Y^A \\ Y_A^\dagger & 0 \end{pmatrix}, \tag{4.1}$$

the bosonic potential can be written in a simple form

$$V_{\text{bos}} \sim \text{Tr}[(\tilde{Y}_A(\tilde{Y}_B\tilde{Y}_B) - (\tilde{Y}_B\tilde{Y}_B)\tilde{Y}_A)^2 - 2(\tilde{Y}_A\tilde{Y}_B\tilde{Y}_C - \tilde{Y}_C\tilde{Y}_B\tilde{Y}_A)^2]. \tag{4.2}$$

Now we assume Y^A are constant Hermite matrices. We further assume that

$$\alpha_{A\ C}^{\ B} \equiv Y_A^\dagger Y^B Y_C^\dagger - Y_C^\dagger Y^B Y_A^\dagger, \tag{4.3}$$

is proportional to the $N \times N$ unit matrix, 1_N, thus they commute with any field. Note that $\alpha_{A\ C}^{\ B}$ is an anti-Hermitian and anti-symmetric under exchange of the indices A and C. Then, we can see from (4.2) that the equations of motion are solved if

$$\alpha_{A\ C}^{\ B} + \alpha_{C\ B}^{\ A} + \alpha_{B\ A}^{\ C} = 0, \tag{4.4}$$

is satisfied. We set $Y^4 = 0$, then A, B, C runs 1 from 3 and the configurations (4.3) with (4.4) may represent a bound state of a M5-brane and M2-branes. Note that by taking the trace of (4.3) and using the relation (4.4), we can see that the configurations (4.3) can not be realized if N is finite, thus we need infinitely many M2-branes, like the D4-D2 bound state in the D2-brane picture.

Because of (4.4), there are 8 independent components of $\alpha_{A\ C}^{\ B}$. These should correspond to the flux on the M5-brane, if there are indeed M5-brane solutions for (4.3) and (4.4). It is very important to find explicit solutions of (4.3) and (4.4) in order to establish these indeed represent the bound state.

5. Conclusions and discussion

In this paper, we have studied the BPS equations of the ABJM action, which will describe the M5-brane. We have found solutions of these equations. These BPS equations are analogues of the Basu-Harvey equation [2] and we expect that the solutions represent N M2-branes ending on the M5-brane. We also discussed the M5-M2 bound state as solutions of the equations of motion, instead of the BPS equation. It is very interesting to investigate the properties of the M5-branes by the solutions.

We can easily extend our study in this paper to some modifications of the ABJM actions, for example, to the orbifold theories [53, 66, 26].

For the Nahm equation and their string theory realization [67, 68], we have an α' exact equivalence between the D2-brane picture (Nahm equation) and the D4-brane picture (Monopole equation) [69] using the tachyon condensation [70]. It is interesting to see how these results are lifted to the M2-brane case.

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